Mathematical League of University of Lodz

Series IV 24/25

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 15.04.25.

Exercise 1. *We have k coins $C_1, C_2, \ldots, C_k$. For all $n \in \mathbb{N}$ the probability that after tossing a coin C_n we will get heads is equal to $\frac{1}{2n+1}$. What is the probability that the number of heads is odd after tossing k coins C_n for $n \in \{1, \ldots, k\}$?*

Exercise 2. *Let $f\colon [a,b] \to \mathbb{R}$ be a function satisfying the following conditions:*

(1) f has the Darboux property, that is, for any $x, y \in [a,b]$ such that $x < y$ and for any $w \in (f(x), f(y))$ (or $w \in (f(y), f(x))$ if $f(x) > f(y)$) there exists $z \in (x,y)$ such that $f(z) = w$.

(2) For any $p \in \mathbb{Q}$ the set $f^{-1}(\{p\})$ is finite.

Show that f is continuous.

Exercise 3. *In a box we have 31 red balls, 41 green balls and 59 blue balls. There are 3 players, each one having a T-shirt of different color: red, green and blue. They sequentially make one of two moves: remove 3 stones of one color from the box, or replace two stones of different colors by two stones of the remaining color. The game finishes when all stones in the box are of the same color. Then the player having the T-shirt of this color wins. Assume that all players play, using the best strategy for them. Can we determine if the game will end and who will be a winner, depending which player starts the game?*