Mathematical League of University of Lodz

Series V 24/25

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address: piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 30.05.25.

Exercise 1. Assume that $f:[0,\infty) \to \mathbb{R}$ is a differentiable bounded function such that $f(x)f'(x) \ge \cos x$ for all $x \in [0,\infty)$. Calculate $\lim_{x\to\infty} f(x)$ or show that this limit does not exist.

Exercise 2. On the chessboard 8×8 there are 8 figures put in such a way that in every row and column there is exactly one figure. Show that figures occupy even number of black squares.

Exercise 3. Let $a_0 = 1$, $a_{2n+1} = a_n$ and $a_{2n+2} = a_n + a_{n+1}$ for $n \ge 0$. Find the set $\{\frac{a_{n-1}}{a_n} : n \in \mathbb{N}\}.$