

Mathematical League of University of Lodz

Series I 23/24

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address:

piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 31.11.23.

Exercise 1. *Let $n \in \mathbb{N}$. Prove that for any $2n$ points on the plane it is possible to join n pairs of points by segments which do not intersect.*

Exercise 2. *Calculate*

$$\lim_{n \rightarrow \infty} ((\sqrt{3} + 1)^n - \lfloor (\sqrt{3} + 1)^n \rfloor),$$

where $\lfloor x \rfloor = \max\{k \in \mathbb{Z}: k \leq x\}$.

Exercise 3. *Find all pairs (n, k) of natural numbers which satisfy*

$$\frac{1}{n} + \frac{1}{k} = \frac{3}{2018}.$$