

Mathematical League of University of Lodz

Series III 23/24

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address:

piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 29.02.24.

Exercise 1. *Show that there are infinitely many sequences of 23 consecutive natural numbers such that sums of squares of all their terms are equal to squares of natural numbers.*

Exercise 2. *A group of $2n$ students wrote a test on which the possible scores were $0, 1, \dots, 10$. Each of these marks occurred at least once, and the average score was equal to $7,4$. Prove that it is possible to divide the group into two subgroups of n students in such a way that the average score for each group was also equal to $7,4$.*

Exercise 3. *Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying*

$$f(xy) = \frac{f(x) + f(y)}{x + y}.$$

for all $x, y \in \mathbb{R}$, with $x \neq -y$. Is there $x \in \mathbb{R}$ such that $f(x) \neq 0$.