

Mathematical League of University of Lodz

Series IV 23/24

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address:

piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 22.04.24.

Exercise 1. Let $x_0, x_1, \dots, y_0, y_1, \dots \in (0, \infty)$ be such that

$$\lim_{n \rightarrow \infty} \frac{x_n}{x_0 + x_1 + \dots + x_n} = \lim_{n \rightarrow \infty} \frac{y_n}{y_0 + y_1 + \dots + y_n} = 0.$$

Let $z_n = \sum_{i=0}^n x_i y_{n-i}$ for $n \in \mathbb{N} \cup \{0\}$. Show that

$$\lim_{n \rightarrow \infty} \frac{z_n}{z_0 + z_1 + \dots + z_n} = 0.$$

Exercise 2. Does there exist a countable set X and an uncountable family F of its subsets, such that for any $A, B \in F$, $A \neq B$ the set $A \cap B$ is finite?

Exercise 3. Let $n \in \mathbb{N} \cup \{0\}$. Calculate

$$\sum_{i=0}^n 2^{n-i} \binom{n+i}{i}.$$