

Mathematical League of University of Lodz

Series V 23/24

For every exercise you can get max. 10. p. Solutions should be delivered on paper (every task on the separate piece of paper) to the room B207 or electronically on the address:

piotr.nowakowski@wmii.uni.lodz.pl. Deadline: 31.05.24.

Exercise 1. Find all pairs (p, q) of prime numbers such that there exist $n, k \in \mathbb{N}$, $k \geq 2$ for which $(p + 1)^q - 1 = n^k$.

Exercise 2. Find all continuous functions $f: \mathbb{R} \rightarrow \mathbb{R}$ satisfying the following equality for all $x, y \in \mathbb{R}$

$$f(x + 2y) = 2f(x)f(y).$$

Exercise 3. Let $n, k \in \mathbb{N}$, $n > k$. Players A and B play a game with n pawns and a board consisting of k squares in one line. At the beginning of the game, the pawns are placed on k leftmost squares. In each turn a player can move any pawn to any free square which is further to the right. The players alternate turns, with player A starting the game. The game ends when the player cannot move (all pawns are on the rightmost squares), and so loses the game. For what n and k player A has a winning strategy, that is, can plan his moves (depending on the moves of his rival) in such a way that he can be sure at the start that he will win no matter of the moves of the rival.